Grade 9

# MATHEMATICS CONTENT BOOKLET: TARGETED SUPPORT

Term 4

# A MESSAGE FROM THE NECT

## NATIONAL EDUCATION COLLABORATION TRUST (NECT)

**Dear Teachers,**
This learning programme and training is provided by the National Education Collaboration Trust (NECT) on behalf of the Department of Basic Education (DBE)! We hope that this programme provides you with additional skills, methodologies and content knowledge that you can use to teach your learners more effectively.

**What is NECT?**
In 2012 our government launched the National Development Plan (NDP) as a way to eliminate poverty and reduce inequality by the year 2030. Improving education is an important goal in the NDP which states that 90% of learners will pass Maths, Science and languages with at least 50% by 2030. This is a very ambitious goal for the DBE to achieve on its own, so the NECT was established in 2015 to assist in improving education and to help the DBE reach the NDP goals.

The NECT has successfully brought together groups of relevant people so that we can work collaboratively to improve education. These groups include the teacher unions, businesses, religious groups, trusts, foundations and NGOs.

**What are the Learning programmes?**
One of the programmes that the NECT implements on behalf of the DBE is the 'District Development Programme'. This programme works directly with district officials, principals, teachers, parents and learners; you are all part of this programme!

The programme began in 2015 with a small group of schools called the Fresh Start Schools (FSS). Curriculum learning programmes were developed for Maths, Science and Language teachers in FSS who received training and support on their implementation. The FSS teachers remain part of the programme, and we encourage them to mentor and share their experience with other teachers. The FSS helped the DBE trial the NECT learning programmes so that they could be improved and used by many more teachers. NECT has already begun this embedding process.

Everyone using the learning programmes comes from one of these groups; but you are now brought together in the spirit of collaboration that defines the manner in which the NECT works. Teachers with more experience using the learning programmes will deepen their knowledge and understanding, while some teachers will be experiencing the learning programmes for the first time.

Let's work together constructively in the spirit of collaboration so that we can help South Africa eliminate poverty and improve education!

www.nect.org.za

# Contents

# INTRODUCTION:
# THREE PRINCIPLES OF TEACHING MATHEMATICS

## PRINCIPLE 1: TEACHING MATHEMATICS DEVELOPMENTALLY

### What is developmental teaching and what are the benefits of such an approach?

- The human mind develops through phases or stages which require learning in a certain way and which influence whether a child is ready to learn something or not.
- If learners are not ready to learn something, it may be due to the fact that they have not reached that level of development yet or they have missed something previously.
- The idea that children's thinking develop from concrete to abstract (Piaget, 1969), was refined (Miller & Mercer, 1993) to include a middle stage, namely the "concrete-representational-abstract" stages. This classification is a handy tool for mathematics teaching. We do not need to force all topics to follow this sequence exactly, but at the primary level it is especially valuable to establish new concepts following this order.
- This classification gives a tool in the hands of the teacher, not only to develop children's mathematical thinking, but also to fall back to a previous phase if the learner has missed something previously.
- The need for concrete experiences and the use of concrete objects in learning, may gradually pass as learners develop past the Foundation Phase. However, the representational and abstract development phases are both very much present in learning mathematics at the Intermediate and Senior Phases.

### How can this approach be implemented practically?

The table on page 7 illustrates how a developmental approach to mathematics teaching may be implemented practically, with examples from several content areas.

### What does this look like in the booklet?

Throughout the booklets, within the topics, suggestions are made to implement this principle in the classroom:

- Where applicable, we suggest an initial concrete way of teaching and learning a concept and we provide educational resources at the end of the lesson plan or topic to assist teachers in introducing the idea concretely.
- Where applicable, we provide pictures (representational/semi-concrete) and/or diagrams (representational/semi-abstract). It may be placed at the clarification of terminology section, within the topic itself or at the end of the topic as an educational resource.
- In all cases we provide the symbolic (abstract) way of teaching and learning the concept, since this is, developmentally speaking, where we primarily aim to be for learners to master mathematics.

## PRINCIPLE 2: TEACHING MATHEMATICS MULTI-MODALLY

### What is multi-modal teaching and what are the benefits of such an approach?

- We suggest that teachers present mathematics topics in three forms to provide for all learners' learning styles and preferences. They (a) explain the idea by speaking about a topic, (b) illustrate it by showing pictures or diagrams and finally (c) present the idea by symbolising it in numbers and mathematical symbols.
- Teaching in more than one form (multi-modal teaching) includes hearing the same mathematical idea in spoken words (auditory mode), seeing it in a picture or a diagram (visual mode) and writing it in a mathematical way (symbolic mode).
- Learners differ in the way they learn and understand mathematical ideas. For one learner it is easier to understand through hearing and for the other through seeing. That is why we open both pathways to the symbolic mode – because here they do not have a choice, they all have to reach that mode, be it through hearing or seeing.

### How can this approach be implemented practically?

The table on page 8 illustrates how a multi-modal approach to mathematics teaching may be implemented practically, with examples from several content areas.

### What does this look like in the booklet?

Throughout the booklets, within the topics at the Senior Phase, we suggest ways to apply this principle in the classroom:

- The verbal explanations under each topic and within each lesson plan, provide the "speak it" or auditory mode.
- The pictures and diagrams give suggestions for the "show it" mode (visual mode).
- The calculations, exercises and assessments under each topic and within each lesson plan, provide the "symbol it" or symbolic mode of representation.

## PRINCIPLE 3: SEQUENTIAL TEACHING

### What is sequential teaching and what are the benefits of such an approach?

- Learners with weak basic skills in mathematics will find future topics increasingly difficult. A solid foundation is required for a good fundamental understanding.
- In order to build a solid foundation in maths, we teach concepts systematically. If we teach concepts out of that order, it can lead to difficulties in grasping concepts.
- Systematic teaching according to CAPS builds progressive understanding and skills.
- In turn, this builds confidence in the principles of a topic before he/she is expected to apply the knowledge and proceed to a higher level.
- We have to continuously review and reinforce previously learned skills and concepts.
- If learners link new topics to previous knowledge (past), understand the reasons why they learn a topic (present) and know how they will use the knowledge in their lives ahead (future), it can help to motivate them and to remove many barriers to learning.

### How can this approach be implemented practically?

If a few learners in your class are not grasping a concept, you as the teacher need to take them aside and teach them the concept again (perhaps at a break or after school).

If the entire class are battling with a concept, it will need to be taught again, however this could cause difficulties in trying to keep on track and complete the curriculum in time.

To finish the year's work within the required time and to also revise topics, we suggest:

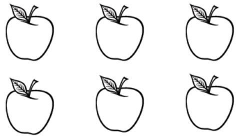
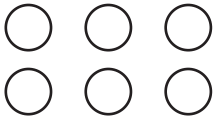
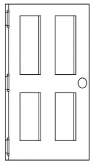
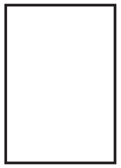
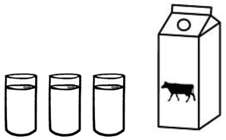
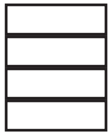
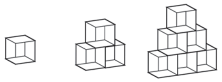
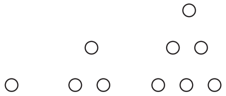
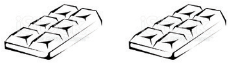
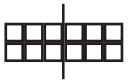
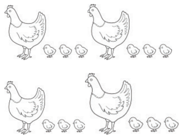
- Using some of the time of topics with a more generous time allocation, to assist learners to form a deeper understanding of a concept, but also to catch up on time missed due to remediating and re-teaching of a previous topic.
- Giving out revision work to learners a week or two prior to the start of a new topic. For example, in Grade 8, before you are teaching Data Handling, you give learners a homework worksheet on basic skills from data handling as covered in Grade 7, to revise the skills that are required for the Grade 8 approach to the topic.

### What does this look like in the booklet?

At the beginning of each topic, there are two parts that detail

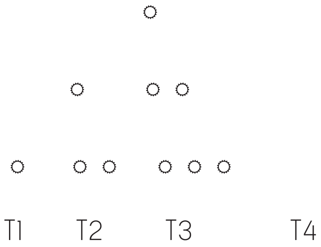
- The SEQUENTIAL TEACHING TABLE lays out the knowledge and skills covered in the previous grade, in the current grade and in the next grade.
- The LOOKING BACK and LOOKING FORWARD summarises the relevant knowledge and skills that were covered in the previous grade or phase and that will be developed in the next grade or phase.

## THREE-STEP APPROACH TO MATHEMATICS TEACHING: CONCRETE-REPRESENTATIONAL-ABSTRACT

| CONCRETE: IT IS THE REAL THING | | REPRESENTATIONAL: IT LOOKS LIKE THE REAL THING | | ABSTRACT: IT IS A SYMBOL FOR THE REAL THING | |
|---|---|---|---|---|---|
| Mathematical topic | Real or physical For example: | Picture | Diagram | Number (with or without unit) | Calculation or operation, general form, rule, formulae |
| Counting | Physical objects like apples that can be held and moved |  |  | 6 apples | $2 \times 3 = 6$ or $2+2+2=6$<br>or $\frac{1}{2}$ of $6 = 3$ or $\frac{2}{3}$ of $6 = 4$ |
| Length or distance | The door of the classroom that can be measured physically |  |  | 80 cm wide<br>195 cm high | Perimeter: $2L + 2W = 390 + 160$<br>$= 550cm$<br>Area: $L \times W = 195 \times 80$<br>$= 15\,600cm^2$<br>$= 1.56m^2$ |
| Capacity | A box with milk that can be poured into glasses |  |  | 1 litre box<br>250 ml glass | $4 \times 250ml = 1\,000ml$<br>$= 1$ litre<br>or<br>1 litre $\div 4 = 0.25$ litre |
| Patterns | Building blocks |  |  | 1; 3; 6… | $n\ (n+1)$<br>$2$ |
| Fraction | Chocolate bar |  |  | 6<br>12 | $6 \quad = 1$<br>$12 \quad 2$<br>or $\frac{1}{2}$ of $12 = 6$ |
| Ratio | Hens and chickens |  | ✱ *** ✱ ***<br>✱ *** ✱ *** | 4:12 | $4 : 12 = 1 : 3$<br>Of 52 fowls ¼ are hens and ¾ are chickens, ie 13 hens, 39 chickens |
| Mass | A block of margarine |  |  | 500g | $500g = 0{,}5\ kg$<br>or calculations like 2 ½ blocks = 1,25kg |

Teaching progresses from concrete -> to -> abstract. In case of problems, we fall back <- to diagrams, pictures, physically.

## MODES OF PRESENTING MATHEMATICS WHEN WE TEACH AND BUILD UP NEW CONCEPTS

| Examples | **SPEAK IT - explain** | **SHOW IT - embody** | **SYMBOL IT - enable** |
|---|---|---|---|
|  | • To introduce terminology<br>• To support auditory learning<br>• To link mathematics to real life | • To help storing and retrieving ideas<br>• To help visual learning<br>• To condense information to one image | • To promote mathematical thinking<br>• To convert situations to mathematics<br>• To enable calculations |
| **IP: Geometric patterns** | "If shapes grow or shrink in the same way each time, it forms a geometric pattern or sequence. We can find the rule of change and describe it in words. If there is a property in the shapes that we can count, each term of the sequence has a number value"<br>"You will be asked to draw the next term of the pattern, or to say how a certain term of the pattern would look. You may also be given a number value and you may be asked, which term of the pattern has this value?**"** |  <br>Question: (a) Draw the next term in this pattern. (b) Describe this pattern. What is the value of the 9th term of this pattern (T9)? **Which term has a value of 120?**<br>To draw up to the 9th term of the sequence and to find out which term has a value of 120, is slow. One is now almost forced to deal with this problem in a symbolic way. | Say out loud:<br>1; 3; 6...<br>1; 3; 6; 10...<br>1; 3; 6;10;15<br>Inspect the number values of terms:<br>T1: 1 = 1<br>T2: 3 = 1+2<br>T3: 6 = 1+2+3<br>T4: 10 = 1+2+3+4<br>T9: 45 = 1+2+3+4+5+6+7+8+9<br>General rule: The value of term n is the sum of n number of consecutive numbers, starting at 1. |
| **SP: Grouping the terms of an algebraic expression** | "We can simplify an algebraic expression by grouping like terms together. We therefore have to know how to spot like terms. Let us say we have to sort fruit in a number of baskets and explain the variables or the unknowns in terms of fruits. Try to imagine the following pictures in your mind:" | Although not in a real picture, we can paint a mind picture to help us understand the principle of classification:<br>• Basket with green apples (a)<br>• Basket with green pears (b)<br>• Basket with green apples and green pears (ab)<br>• Basket with yellow apples ($a^2$)<br>• Basket with yellow apples and green pears ($a^2b$)<br>Or in diagram form<br>a □ b ○ ab □○<br>$a^2$ ■ $a^2b$ ■○ | Group and simplify the following expression:<br>$4b - a^2 + 3a^2b - 2ab - 3a + 4b + 5a - a - 2ab + 2a^2b + a^2b$<br>$= -3a + 5a - a + 4b + 4b - 2ab - 2ab - a^2 + 3a^2b + 2a^2b + a^2b$<br>$= a + 8b - 4ab - a^2 + 6a^2b$ |

# TOPIC 1: TRANSFORMATION GEOMETRY

## INTRODUCTION

- This unit runs for 9 hours.
- It is part of the Content Area, ‘Space and Shape’ and counts for 30% in the final exam.
- The unit covers transformations of points and shapes on the Cartesian plane.
- Learners must be encouraged to describe any transformation in two parts – the type of transformation as well as the more specific explanation.

  For example: A reflection about the y-axis.
- It is important to note that this topic gives learners the opportunity of gaining a deeper understanding of functions for future grades.

## SEQUENTIAL TEACHING TABLE

| INTERMEDIATE PHASE/GRADE 8 | GRADE 9 | GRADE 10/FET PHASE |
|---|---|---|
| **LOOKING BACK** | **CURRENT** | **LOOKING FORWARD** |
| • Recognise, describe and perform transformations with points on a coordinate plane, focusing on reflections of a point about the  and y-axes and translating a point within and across quadrants<br>• Recognise, describe and perform transformations with triangles on a coordinate plane, focusing on the coordinates of the vertices when reflecting a triangle in the axes, translating a triangle across the quadrants and rotating a triangle about the origin<br>• Use proportion to describe the effect of enlargement and reduction on area and perimeter of geometric figures | • Recognise, describe and perform transformations with points, line segments and simple geometric figures on a coordinate plane, focusing on reflections of a point about the  and y-axes; translating a point within and across quadrants; and reflection in the line<br>• Use proportion to describe the effect of enlargement and reduction on area and perimeter of geometric figures<br>• Investigate the coordinates of the vertices of figures that have been enlarged or reduced by a given scale factor | • Transformations are not covered again in CAPS. However, it is important to note that skills gained in this section of mathematics will be useful in the following areas covered in the FET phase:<br>• Functions<br>• Analytical geometry<br>• Trigonometry<br>• Euclidean Geometry<br>• Differential Calculus |

# GLOSSARY OF TERMS

| Term | Explanation |
|---|---|
| **Translation** | A horizontal and/or vertical slide from one position to another.<br>Every point moves the same distance and in the same direction.<br>The translated shape is congruent to the original shape. |
| **Reflection** | A mirror image of a shape. In transformation geometry, reflections are done across the $x$-axis or the $y$-axis in the Cartesian Plane.<br>Every point is the same distance from a central line.<br>The reflected shape is congruent to the original shape. |
| **Rotation** | A turning of a shape around a certain point. In this section it is always around (from) the origin on a Cartesian Plane.<br>Every point makes an imaginary circle from the point of rotation.<br>The rotated shape is congruent to the original shape. |
| **Congruent** | Exactly the same size. All sides and all angles are equal. |
| **Similar** | Same shape but different size. Sides change length but all angles remain equal. |
| **Enlargement** | The resizing of a shape to make it bigger.<br>The shape of the transformed shape will be similar to the original shape. |
| **Reduction** | The resizing of a shape to make it smaller.<br>The shape of the transformed shape will be similar to the original shape. |
| **Scale factor** | The amount by which the object is changed. |

# SUMMARY OF KEY CONCEPTS

Learners need to have an understanding that:

- Translations, reflections and rotations only change the position of a figure and these figures will be congruent (exactly the same size).
- Enlargements and reductions change the size of a figure but they will be in proportion to the original shape and will therefore be similar.

Note: When teaching this section, it is advisable to start again with just transforming points in the Cartesian plane as done in Grade 8. As a teacher, it may be a good idea to go through this section in the Grade 8 booklet again in order to consolidate your understanding

**Teaching Tip:**
Remind learners what the word congruent means from a mathematical point of view: Congruent shapes have equal angles and equal sides.

## Translations

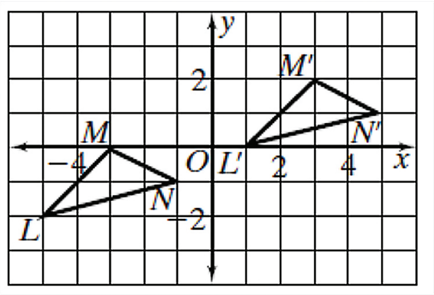
This is the moving of the point or figure either horizontally, vertically or a combination of the two.

**Vertical Translations: $A(x;y) \rightarrow A'(x;y+q)$**
If q is positive we would move the point(s) up the number of units represented by q. If q is negative the point(s) move down the units required.

**Horizontal Translations: $A(x;y) \rightarrow A'(x+p;y)$**
If p is positive we would move the point(s) right the number of units represented by p. If p is negative the point (s) move left the units required.

The point(s) may need to be moved both vertically and horizontally. This would be represented by:
$A(x;y) \rightarrow A'(x+p;y+q)$

If we are transforming the point(s) we indicate the image of the point(s) by adding an apostrophe to the letter that represents the point(s)



∆LMN has been translated 6 units to the right and 2 units up to form ∆L'M'N'

Learners need to gain an understanding that for translations:

- to the right or left, the $x$-value changes while the $y$-value stays the same
- up or down, the $y$-value changes while the $x$-value stays the same

### Reflections

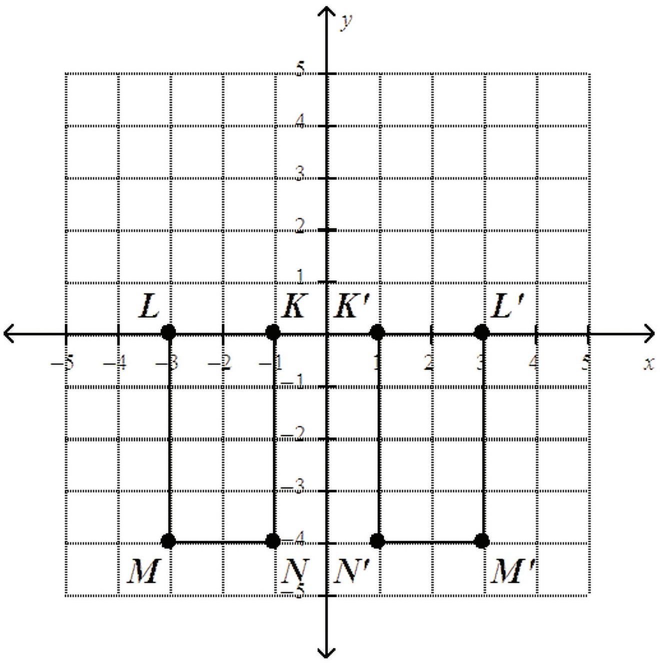
This is an image reflected over an axis of symmetry. The resulting image is a mirror image of the original.

1. Reflection over the $y$-axis ($x = 0$): A($x; y$)→A' ($-x; y$)

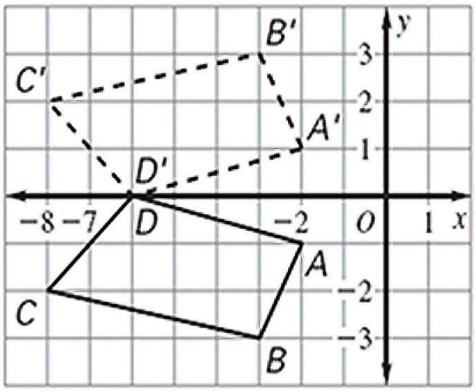
For example: Rectangle KLMN has been reflected in the y-axis:

K (–1 ; 0) L (–3 ; 0) M (–3 ; -4) N (–1 ; –4)

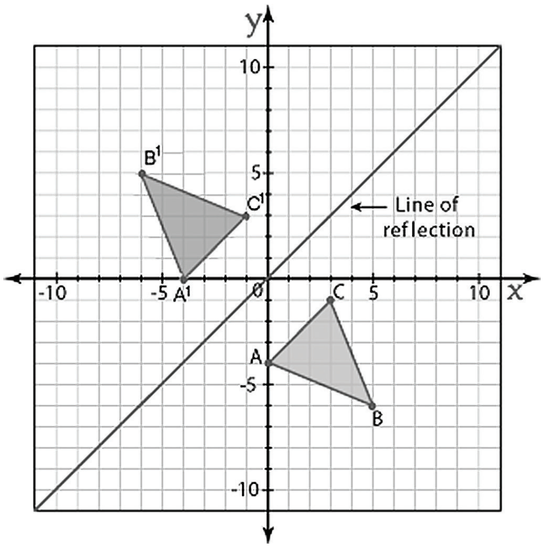
K’ (1 ; 0) L’ (3 ; 0) M’ (3 ; –4) N’ (1 ; –4)



2. Reflection over the $x$-axis ($y = 0$): A($x; y$)→A' ($x; -y$)
For example: Quadrilateral ABCD has been reflected in the $x$-axis:
A (–2 ; –1) B (–3 ; –3) C (–8 ; –2) D (–6 ; 0)
A' (–2 ; 1) B' (–3 ; 3) C' (–8 ; 2) D' (–6 ; 0)



3. Reflection over the line $y = x$: A ($x ; y$) → A' ($y ; x$)
For example: Triangle ABC has been reflected in the line $y = x$:
A (0 ; –4) B (5 ; –6) C (–3 ; –1)
A' (–4 ; 0) B' (–6 ; 5) C' (–1 ; –3)



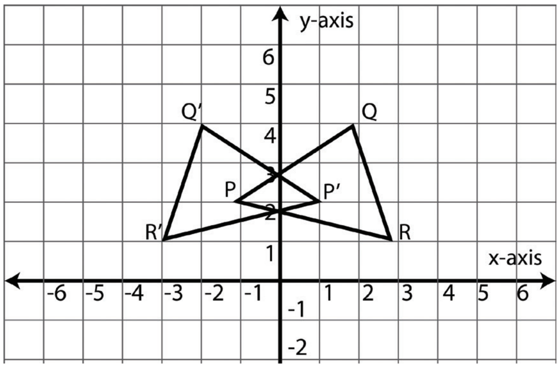
**Teaching Tip:** Once a reflection has been performed, encourage learners to take their ruler and place it on point A and point A' and check the following:

- Is the ruler lying at a right angle to the line of reflection?
- Is the original point and its reflected point exactly the same distance from the line of reflection?

Note: If a shape crosses over the axis that it is being reflected in, in its original form then when it has been reflected parts of it will overlap the original shape.

For example:
△PQR is reflected in the $y$-axis:

| | | |
|---|---|---|
| P ( -1 ; 2 ) | Q ( 2 ; 4 ) | R ( 3 ; 1 ) |
| P' ( 1 ; 2 ) | Q' ( -2 ; 4 ) | R' ( -3 ; 1 ) |



Learners need to gain an understanding that for reflections:

- in the $y$-axis, the $x$-value changes sign while the $y$-value stays the same
- in the $x$-axis, the $x$-value stays the same while the $y$-value changes signs
- in the line $y = x$, the $x$-value and $y$-value are interchanged

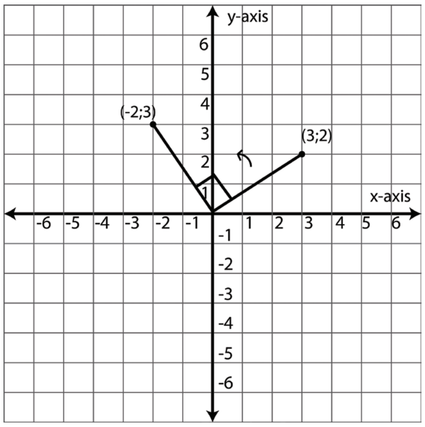
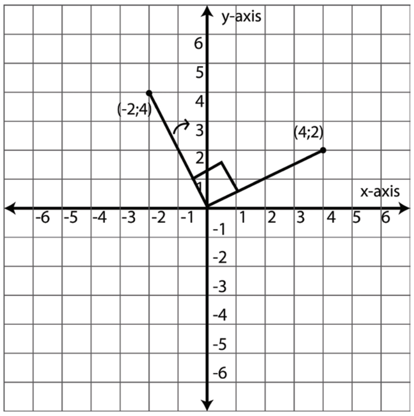
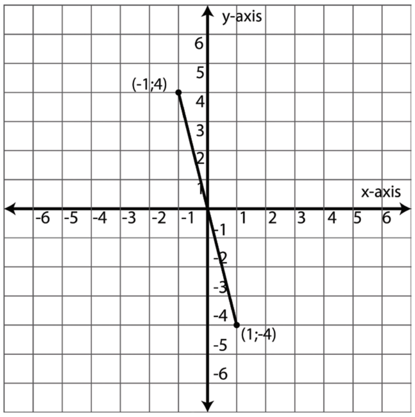
## Rotations

Although the focus in Grade 9 is not on rotations it is worth revising the following from Grade 8.

Before the rotating of points are discussed, please note that, similarly to grade 8, learners should be encouraged to establish the rules for themselves once they have experimented and observed what occurs to the coordinate when it has been rotated. Learners should NOT be taught the rules and asked to learn them.

(The explanation/rule on the right hand side of the following table are for the teacher's information only in order to create a better understanding for the teaching of this section)

Rules for rotating:

<table>
<tr><td><b>90º anti-clockwise</b></td><td>The coordinates will swop over and the new x-coordinate (which was the y-coordinate) will change signs<br>$(x \,;\, y)$ $(-y \,;\, x)$<br>( 3 : 2 ) ( -2 : 3 )</td></tr>
<tr><td colspan="2">
</td></tr>
<tr><td><b>90º clockwise</b></td><td>The coordinates will swop over and the new y coordinate (which was the x-coordinate) will change signs<br>$(x \,;\, y)$ $(y \,;\, -x)$<br>( -2 : 4 ) ( 4 : 2 )</td></tr>
<tr><td colspan="2">
</td></tr>
<tr><td><b>180º</b></td><td>The signs of each coordinate will change<br>$(x \,;\, y)$ $(-x;\, -y)$<br>( -1 : 4 ) ( 1 : - 4 )</td></tr>
<tr><td colspan="2">
</td></tr>
</table>

## Enlargement and reduction through the origin

This is the changing of the size of the original object. The resulting object will remain similar to the original. The changes will be proportionate and thus the shapes will be similar.

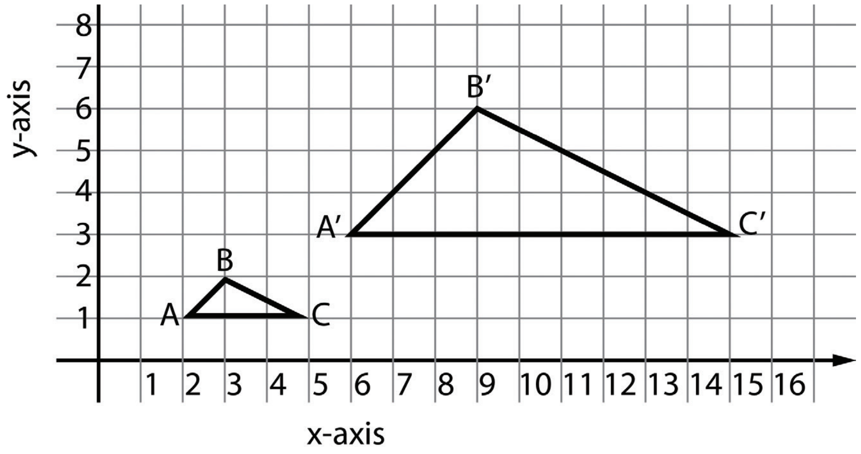
**Teaching Tip:**
Remind learners what the word similar means from a mathematical point of view: Similar shapes have equal angles and sides are in proportion.

**1. Enlargement by factor k: A(x;y)→A' (kx;ky)**

We multiply the $x$ and $y$ values of the coordinate by the factor that we wish to enlarge the object by.

For example:
Triangle ABC has been enlarged by a scale factor of 3:

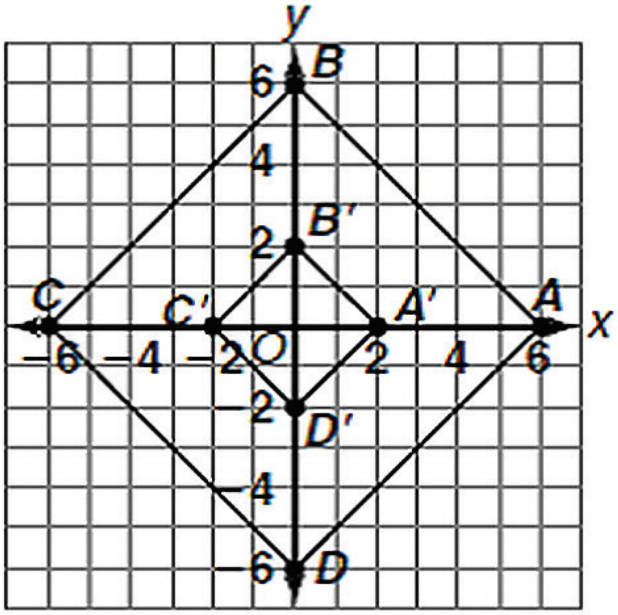
A (2 ; 0)    B (3 ; 2)    C (5 ; 1)
A’ (6 ; 0)    B’ (9 ; 6)    C’ (15 ; 3)    Note that each coordinate was multiplied by 3



**2. Reduction by factor k:** $A(x;y) \rightarrow A^{'}\left(\frac{1}{k}x\,;\frac{1}{k}y\right)$

We still have to multiply as we always multiply by scale factor, but because we are reducing we multiply by a fraction where the factor becomes the denominator of the fraction.

For example:
ABCD has been reduced by a scale factor of 3
A (6 ; 0)    B (0 ; 6)    C (-6 ; 0)    D (0 ; -6)
A’ (2 ; 0)    B’ (0 ; 2)    C’ (-2 ; 0)    D’ (0 ; -2)
Note that each coordinate was multiplied by $\frac{1}{3}$



Scale factor – is: $\frac{image\ length}{pre-image\ length}$

- If the scale factor is greater than 1, the figure becomes larger.
- If the scale factor is between 0 and 1, the figure becomes smaller.

# TOPIC 2: GEOMETRY OF 3D OBJECTS

## INTRODUCTION

- This unit runs for 9 hours.
- It is part of the content area, ‘Space and Shape’ and counts for 30% in the final exam.
- The unit covers the classification and building of 3D objects
- The purpose of needing a good knowledge of 3D objects is simply that we live in a three-dimensional world. Every object you can see or touch has three dimensions that can be measured: length, width and height. The room you are sitting in can be described by these three dimensions.

## SEQUENTIAL TEACHING TABLE

| INTERMEDIATE PHASE/GRADE 8 | GRADE 9 | GRADE 10/ FET PHASE |
|---|---|---|
| LOOKING BACK | CURRENT | LOOKING FORWARD |
| • Describe, name and compare the 5 platonic solids in terms of shape and number of faces; number of vertices and number of edges<br>• Use nets to create models of geometric solids, including cubes, prisms and pyramids | • Revise properties of the 5 platonic solids in terms of shape and number of faces; number of vertices and number of edges<br>• Recognise and describe the properties of spheres and cylinders<br>• Use nets to create models of geometric solids, including cubes, prisms, pyramids and cylinders | Measurement becomes part of Euclidean geometry in the FET phase.<br>Learners will be required to:<br>• Solve problems involving volume and surface area of solids studied in earlier grades as well as spheres, pyramids and cones and combinations of those objects. |

# GLOSSARY OF TERMS

| Term | Explanation / Diagram |
|---|---|
| **2D** | 2-dimensional<br>A shape that is made up of length and width (2 dimensions). |
| **3D** | 3-dimensional<br>A shape that is made up of length, width and height (3 dimensions). |
| **Congruent** | Exactly the same size. |
| **Polygon** | A 2D shape in which all the sides are made up of line segments. A polygon is given a name depending on the number of sides it has.<br>For example: A 5 sided polygon is called a pentagon. |
| **Solid** | An object that occupies space (3-dimensional). |
| **Platonic Solid** | A solid shape where all the faces are congruent and all the edges are the same length. The cube is the most common platonic solid.<br>Other platonic solids: tetrahedron (4 triangular faces); octahedron (8 triangular faces); icosahedron (20 triangular faces); dodecahedron (12 pentagonal faces). |
| **Polyhedron** | A solid in which all the surfaces (faces) are flat. |
| **Prism** | A solid with parallel equal bases. The bases are both polygons. |
| **Right Prism** | A prism which has the sides at right angles to the base. |
| **Face** | A flat surface of a prism. |
| **Edge** | Where the faces of a prism meet. |
| **Vertex** | Where the edges of a prism meet (the corner). |
| **Cube** | A solid with six equal square faces. |
| **Rectangular Prism** | A solid with six rectangular faces. |
| **Triangular prism** | A solid with two equal triangular faces (one is the base) and three rectangular faces. |
| **Cylinder** | A solid with two equal circular faces (one is the base) and one rectangle (curved). |
| **Net** | A 2D shape, that when folded forms a 3D shape. |
| **Pyramid** | A solid object where the base is a polygon and the side faces are all triangles that meet at an apex (point).<br>Pyramids are named according to their base, for example, square pyramid or triangular pyramid. |
| **Apex** | A point where the edges of a pyramid meet.<br>The point at the top of a pyramid. |

# SUMMARY OF KEY CONCEPTS

## Classifying 3D objects – solids and platonic solids

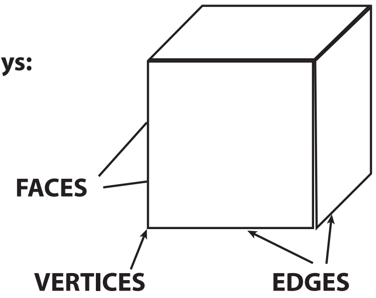
1. Solids

   All 3D objects occupy space and are therefore known as solids.
   Every 3-dimensional solid is made up of faces, edges and vertices.

### FACES, VERTICES and EDGES

**3D shapes can be described in 3 ways:**

**Faces - the sides of the shape**
**Vertices - the corners**
**Edges - where the faces meet**



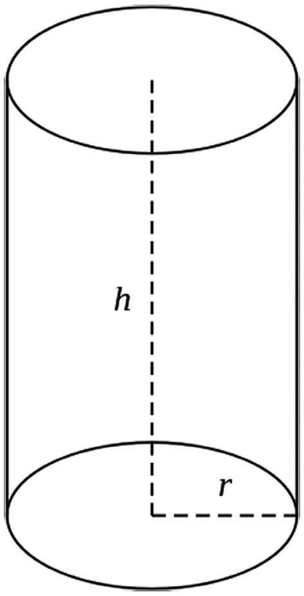
**Teaching Tip:** Use as many manipulatives as possible (ask learners to bring 3D objects from home) and spend some time allowing the learners to hold the objects and count the faces, edges and vertices.

The two newest solids to focus on in grade 9:

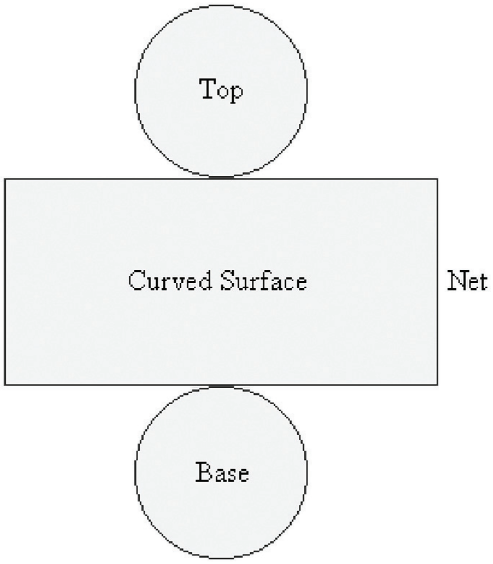
a. The cylinder

   A cylinder has a circular base and the sides are at right angles to the base. This makes the cylinder a right prism.

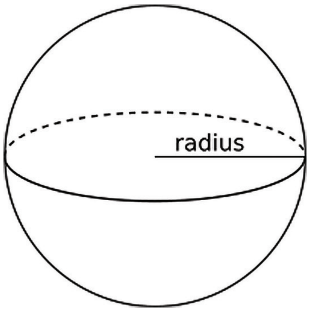
   Remind learners that they have already worked extensively with the cylinder in Term 3 when they found the surface area and the volume of it.



The net of a cylinder:

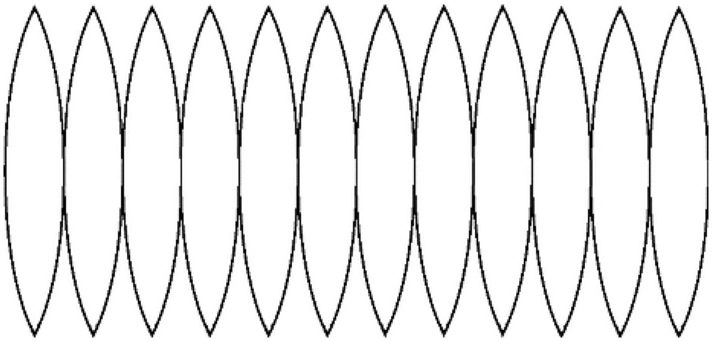


b. The sphere
A sphere is a 3D object where each point found on the surface is exactly the same distance from the centre of the sphere.



The net of a sphere is not very easy to visualize.

**Teaching Tip:** If possible, bring an orange and a knife to school. Cut into the skin of the orange from the top of the orange to the bottom a number of times going all the way around the orange. Peel each portion off to show why the net of a sphere looks like this:



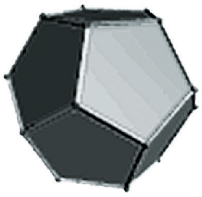
If you have access to the internet, there is a link to a youtube video on the resource page.

### 2. Platonic solids


Tetrahedron
4 faces
4 vertices
6 edges


Cube
6 faces
8 vertices
12 edges


Octahedron
8 faces
6 vertices
12 edges

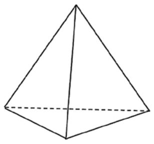
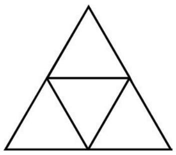
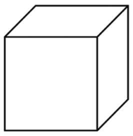
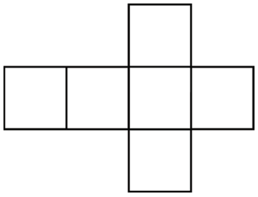
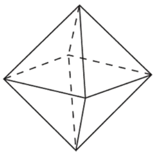
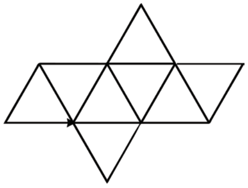
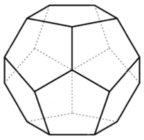
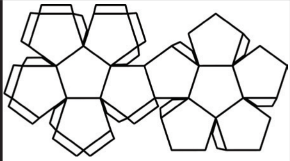
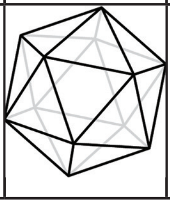
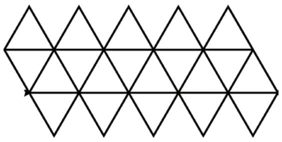

Dodecahedron
12 faces
20 vertices
30 edges


Icosahedron
20 faces
12 vertices
30 edges

A polyhedron is regular if all its faces are equal polygons and the same number of faces meet at every vertex

These are the only 5 3D objects that have these characteristics.

The following table summarises the main characteristics of each platonic solid and also shows the net of each one.

| Platonic Solid | Picture | Number of Faces | Shape of Faces | Number of Faces at Each Vertex | Number of Vertices | Number of Edges | Unfolded Polyhedron (net) |
|---|---|---|---|---|---|---|---|
| Tetrahedronc |  | 4 | Equilateral Triangle (3-sided) | 3 | 4 | 6 |  |
| Cube |  | 6 | Sqare (4 Sided) | 3 | 8 | 12 |  |
| Octahedron |  | 8 | Equilateral Triangle (3-sided) | 4 | 6 | 12 |  |
| Dodecahedron |  | 12 | Regular Pentagon (5-sided) | 3 | 20 | 30 |  |
| Isocahedron |  | 20 | Equilateral Triangle (3-sided) | 5 | 12 | 30 |  |

## Classifying 3D objects – Euler's law

Euler's law states the following:
For any polyhedron (a solid figure with many plane faces, typically more than six) the:

- number of faces
- plus the number of vertices (corners)
- minus the number of edges always equals 2

This can be written: **F + V − E = 2**

For example, consider the triangular prism. It has 5 faces, 6 vertices and 9 edges

$$5 + 6 - 9 = 2$$

Using a table similar to the one above (most textbooks will have one in them otherwise a blank table is available at the end of the topic in the resource section) encourage learners to find Euler's law for themselves by completing the table and answering the questions.

## Building 3D models – using and constructing nets

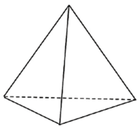
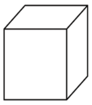
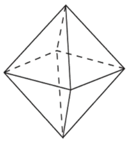
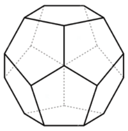
Learners need to be encouraged to deconstruct 3D shapes to find the nets for themselves. This topic will be covered in a number of lessons and there should be plenty of time to allow the learners to 'play' and discover for themselves.

Encourage learners to bring the inner part of a toilet roll to school and a box that has had cereal, biscuits or something similar in and allow them to 'undo' it and see what net was used to make the 3D object.
Ask teachers of other subjects to bring in items from home to help you make up a reasonable collection of a variety of boxes etc so all learners can take part even if they don't all contribute by bringing their own.

A few nets are available in the resource section on the following pages.

# RESOURCES

Complete the following table then answer the questions:

| Platonic solid | Number of Faces | Number of Vertices | Number of Edges |
|---|---|---|---|
| <br>Tetrahedron | | | |
| <br>Hexahedron (Cube) | | | |
| <br>Octahedron | | | |
| <br>Dodecahedron | | | |
| <br>Icosahedron | | | |

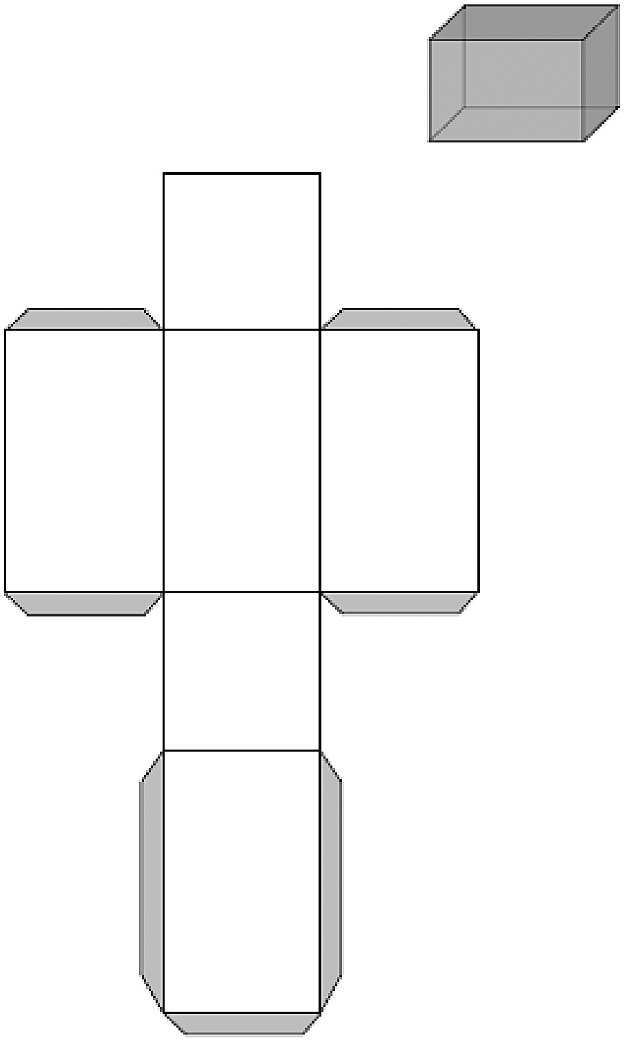
1. Look at the first shape in the table above. Find the sum of the number of faces and the number of vertices. How does this total compare with the number of edges?
2. Now look at the other shapes in the table. Compare the sum of the number of faces and vertices with the number of edges. What did you find out? Is there a rule for all of the shapes?
3. So what is Euler's formula?

Sphere of a net demonstrated using an orange video link:

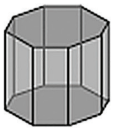
https://www.youtube.com/watch?v=SrDASzRJDPA

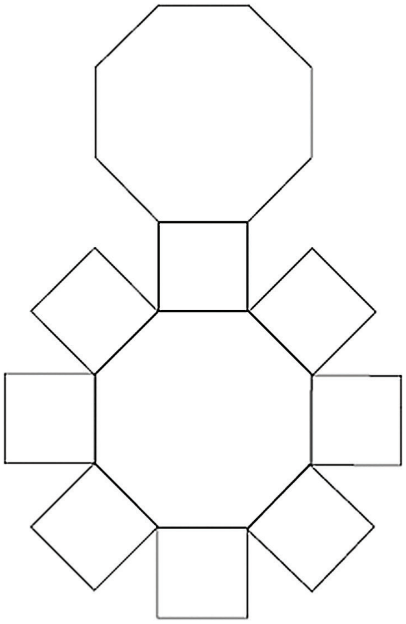
## NETS

### Rectangular Prism

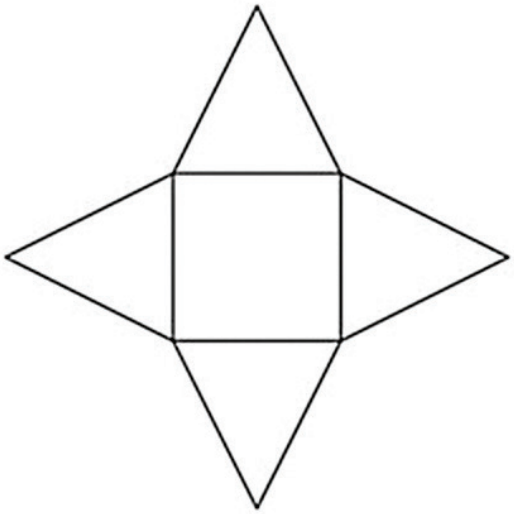


## Octagonal Prism



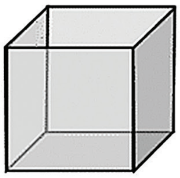
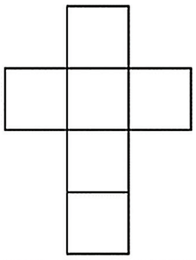
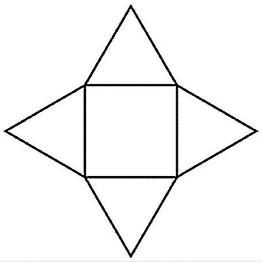
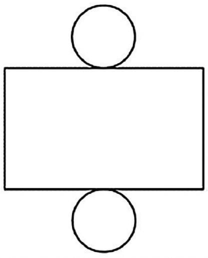
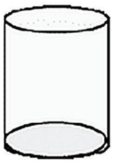
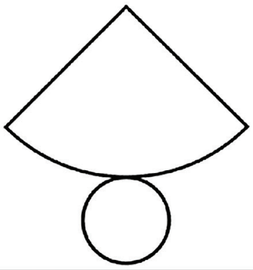
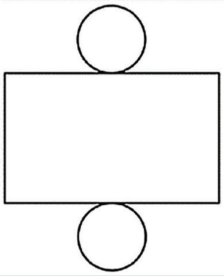
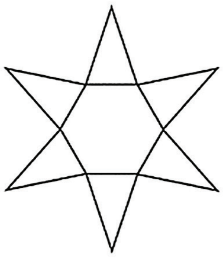
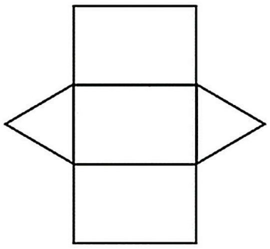
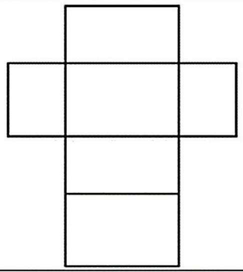
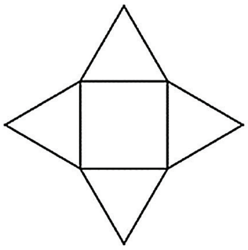
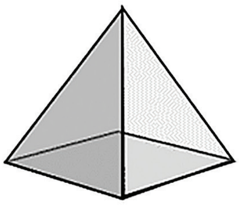
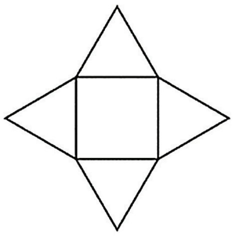
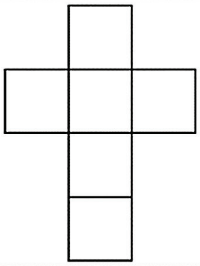
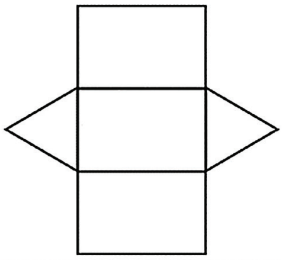
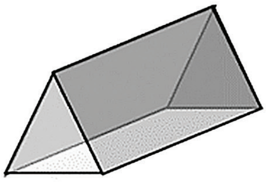
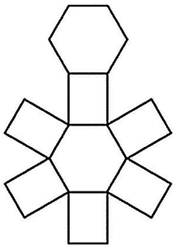
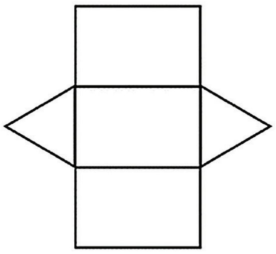
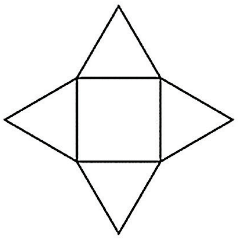


## Square Pyramid



If photocopying is possible, a worksheet similar to these are useful for learners who need remediation and are struggling.

## For each 3d shape, shade the correct net.

| | | | |
|---|---|---|---|
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

Join each shape to the matching net.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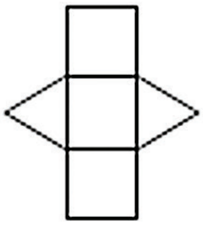

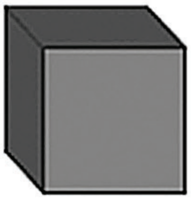
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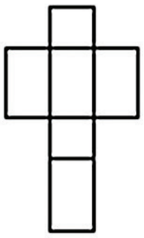


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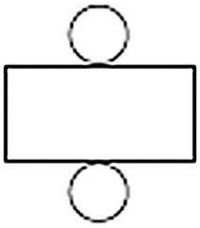


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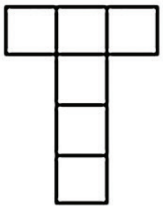


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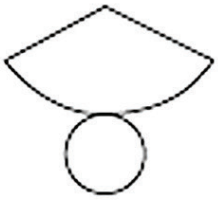


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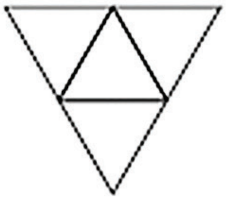


# TOPIC 3: DATA HANDLING

## INTRODUCTION

- This unit runs for 10,5 hours, which includes: Collect, organize and summarize data (4 hours); Represent data (3 hours) and Interpret, analyse and report data (3,5 hours)
- It counts for 10% in the final exam.
- The unit covers all aspects of data handling as laid out above
- It is important to note that learners need to be exposed to a variety of contexts that deal with social and environmental issues. Learners also need to practice collecting, organising, representing and analysing data. Time needs to be spent discussing and showing learners the differences between the different types of graphs and when one may be more useful than the other.
- If you have access to computers, the learners could use them to draw some of the graphs in Excel.
- The purpose of teaching data handling is to provide learners with the knowledge of how data is collected and the ways in which it can be represented.
- Surveys, graphs and charts are often used by the media to inform, persuade and at times, mislead the audience. Learners need to be made aware of this.
- Data handling is an important section of mathematics as it used in research. Tertiary education makes extensive use of research.

# SEQUENTIAL TEACHING TABLE

| INTERMEDIATE PHASE/GRADE 8 | GRADE 9 | GRADE 10/FET PHASE |
|---|---|---|
| LOOKING BACK | CURRENT | LOOKING FORWARD |
| • Collect data<br>• Pose questions relating to social, economic, and environmental issues<br>• Select appropriate sources for the collection of data<br>• Distinguish between samples and populations<br>• Design and use simple questionnaires to answer questions with multiple choice responses<br>• Organize and record data<br>• Group data into intervals<br>• Summarize and distinguish between ungrouped numerical data by determining measures of central tendency and measures of dispersion<br>• Summarize data in short paragraphs that include the role of extremes in the data | • Collect data<br>• Pose questions relating to social, economic, and environmental issues<br>• Select and justify appropriate sources for the collection of data<br>• Distinguish between samples and populations<br>• Select and justify appropriate methods for collecting data<br>• Organize numerical data in different ways in order to summarize by determining measures of central tendency, measures of dispersion, including extremes and outliers<br>• Organize data according to more than one criteria<br>• Critically read and interpret data<br>• Critically compare two sets of data related to the same issue<br>• Critically analyse data<br>• Summarize data in short paragraphs that include the role of extremes and outliers in the data | • Collect, organise and interpret data in order to determine measures of central tendency; five number summary; box and whisker diagrams; and measures of dispersion.<br>• Represent measures of central tendency and dispersion by: using ogives; and calculating the variance and standard deviation and representing results graphically.<br>• Represent skewed data in box and whisker diagrams, and frequency polygons. Identify outliers<br>• Represent bivariate numerical data as a scatter plot and suggest whether a linear, quadratic or exponential function would best fit the data.<br>• Calculate the linear regression line<br>• Calculate the correlation co-efficient |

## GLOSSARY OF TERMS

| Term | Explanation |
|---|---|
| **Data** | Facts or information collected from people or objects. Data is plural for datum. |
| **Population** | The entire group of people or objects that data is being collected from |
| **Sample** | A smaller part of the population if the population is too large |
| **Random** | How to choose a smaller sample of the population to attempt to not be biased |
| **Questionnaire** | A set of printed questions with a choice of answers used in the data collection process |
| **Survey** | The collecting of data from a group of people |
| **Discrete data** | Data that can only take certain values. For example, the number of learners in a class (there can't be half a learner) |
| **Continuous data** | Data that can take on any value within a certain range. For example, the heights of a group of learners (heights could be measured in decimals) |
| **Tally** | A way of keeping count by drawing marks. Every fifth mark is drawn across the previous four (to form a gate-like diagram) so you can easily see groups of five. |
| **Frequency tables** | A table that lists a set of scores and their frequency. Often used with tallies. It summarises the totals and shows how often something has occurred. |
| **Stem-and-leaf tables** | A way to summarise data by splitting each value (e.g. 26) into a stem (2) and a leaf (6) |
| **Mean** | The average of a set of numbers. Calculated by adding all the values then dividing by how many numbers there are. |
| **Median** | The middle number in a sorted list of numbers. To find the median, place all numbers in order from smallest to biggest and find the middle number. |
| **Mode** | The number that appears the most often in a set of data. There can be two modes. There could also be no mode in a set of data. |
| **Range** | The difference between the highest and lowest value in a set of data. |
| **Bar Graphs** | A graph drawn using rectangular bars to show how large each value is. The bars could be horizontal or vertical |
| **Double bar graphs** | A double bar graph displays information using two bars beside each other comparing two sets of data linked to the same thing. Example: A school's results over two years would form the double bar and a different school's results could be in a separate double bar next to it. |
| **Histograms** | A graph representing data that is grouped into ranges and each bar represents data that follows on from the previous bar. Example, one bar could represent how many learners got a mark from 40-49 and the bar immediately next to it would represent 50-59. |
| **Pie chart** | A circular chart divided into sectors. Each sector represents an amount of something relative to its size. |
| **Broken line graph** | A graph that uses points connected by lines to show how something changes in value as time goes by or as something else happens. |
| **Scatter plots** | A graph in which the values of two variables are plotted along two axes. The pattern of the resulting points reveals whether there is any correlation between the two sets of values. |
| **Outliers** | These are values that are significantly higher or lower than all the other values in the data set. They are also called extremes. Outliers can affect the mean of the data and are sometimes excluded when calculations are done. |

# SUMMARY OF KEY CONCEPTS

## Collecting Data

1. Data can be collected in a number of ways. A questionnaire is often used. In order to make the analysing of the data easier after having people fill out a questionnaire, it needs to be carefully thought out. It is best to give multiple choice answers instead of open ended questions.

   For example:

   | | | | | | | | | | |
   |---|---|---|---|---|---|---|---|---|---|
   | Gr 8 | | Gr 9 | | Gr 10 | | Gr 11 | | Gr 12 | |

   What Grade are you in? (Place a cross next to your grade)

   The response to a question similar to the one above could also be collected verbally and a tally table could be used as the responses are given.
2. A difficult question to analyse would be: What do you think about the new school tuckshop? There would be a large variety of answers making it more difficult to organise and analyse.
3. In order to collect data, the population needs to be considered. If a person wanted to do a survey amongst teenagers for example, it will be impossible to survey all the teenagers in the country or even a school. Therefore, a sample of your population (teenagers) would be needed. A random sample is the better way to choose a sample so that each person in the population has an equal opportunity to be part of the survey. This could be done by taking class lists and picking every fifth learner. It would be a biased sample if a person just chose their friends.

**Teaching Tip:**
Take some time to discuss the difference between a population and a sample with the learners. Discuss a few situations and ask learners who the population is and how they would go about choosing a sample of the population. A discussion of the word biased would also be appropriate at this stage.

## Organizing and Summarizing Data

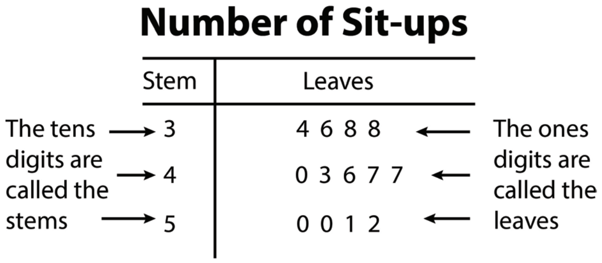
1. Once the data has been collected, it needs to be organised and recorded.
2. Tally and frequency tables could be used at this stage or if a computer is available, all the data could be put into an excel spreadsheet (excel can also create graphs for you from the data).

   For example: If you had done a survey on learner's favourite colours, a tally table with the frequency recorded may look like this:
3. Stem-and-leaf plots could also be used to summarise a set of data.

| | | |
|---|---|---|
| Yellow | 𝍷𝍷𝍷𝍷 | 4 |
| Red | 𝍸 | 5 |
| Blue | 𝍸 𝍷 | 6 |
| Green | 𝍷 | 1 |
| Pink | 𝍷𝍷𝍷𝍷 | 4 |

The 'stem' represents the 10's (or 100's) part of the number and the 'leaves' represent the units.

For example: The stem-and-leaf plot below represents the number of sit-ups done by 8 athletes. The 'leaf' side should always be in numerical order. That way, it is easy to find the median if necessary.



Key: 3 | 6 = 36

4. In order to summarise the data, it is good to look at the measures of central tendency and the measures of dispersion.
5. Measures of central tendency. A measurement of data that indicates where the middle of the information lies. All the data is described by tending towards just one number. A measure of central tendency is a single value that describes the way in which a group of data cluster around a central value. It is a way to describe the centre of the data set. There are three measures of central tendency: mean, median and mode

For example: The following numbers represent the shoe sizes of some learners in your class

8 6 6 5 7 9 8 6 5 8 9 6 7

To find the:

a. Mean

Add all the values and divide by the number of values

$$\frac{90}{13} = 6,92$$

b. Median

Arrange the values from smallest to biggest and find the middle value. (If there is an even amount of data, add the two middle values and divide by two – in other words find the mean of the two middle numbers)

5 5 6 6 6 6 7 7 8 8 8 9 9

The median is 7

c. Mode

Find a number that appears most often. There can be two modes – if this is the case we call the set of data bimodal. The mode is 6. It is also possible that a set of data has no mode.

Mean is the most common form used to find one value to represent a set of data. It is considered a good measure at it takes all the data into account. However, it also has its limitations – if there is an outlier (an extreme value which is much higher or much lower than the other values) then it isn't always a good measure. It also can't be used if the data is not numerical – for example if your data is a list of colours like in the above example of a stem-and-leaf plot)

6. Measures of dispersion. This shows how spread out the data is. There are many measures of dispersion which will be covered in later grades but this year the only one required is 'range'.
   To find the range, subtract the lowest value from the highest value.
   From the shoe sizes set of data:

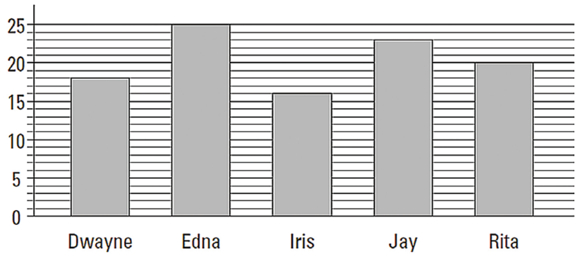
   9 – 5 = 4

   4 is the range

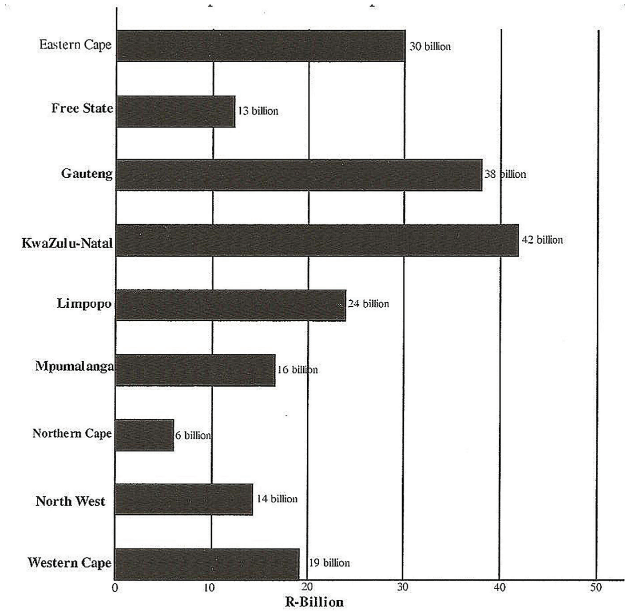
7. Extremes (or outliers) need to be taken into account when making any of the above calculations. If one or even two sets of data are very different to the rest of the data they can make a significant difference to whether a measure of central tendency (and in particular the mean) or the measure of dispersion is considered reliable or not. The affected mean or range incorrectly displays a bias toward the outlier value. The median and mode values, which express other measures of central tendency, are largely unaffected by an outlier.

## Representing Data

1. Once the data has been organised and summarised it needs to be represented in a visual way that is easy to read and understand.
2. Graphs are the common choice to represent data. The visual representation makes it easier for a person to understand the data collected rather than a long wordy explanation.
3. There are two types of data and each one lends itself to being represented using a different graph. Discrete data has clear separation between the different possible values, while continuous data doesn't. We use bar graphs for displaying discrete data, and histograms for displaying continuous data.
4. Graphs and examples:
   a. Bar Graphs

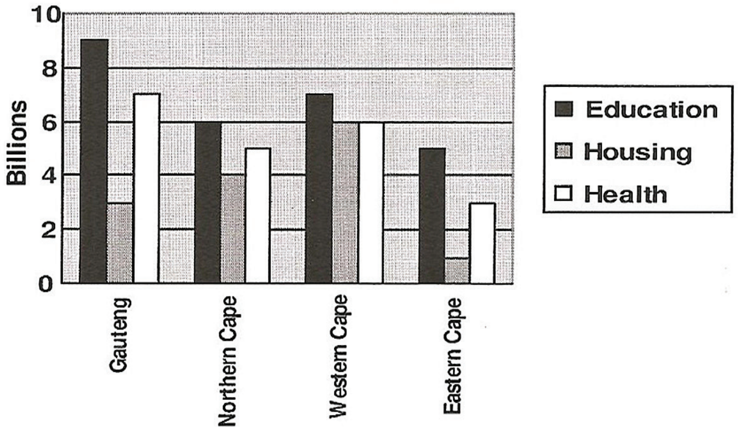


A bar graph is used to show discrete data. This graph shows the number of sweets children got in a special lucky dip.
Notice that each bar is kept separate from the other.
A bar graph can have vertical or horizontal bars.



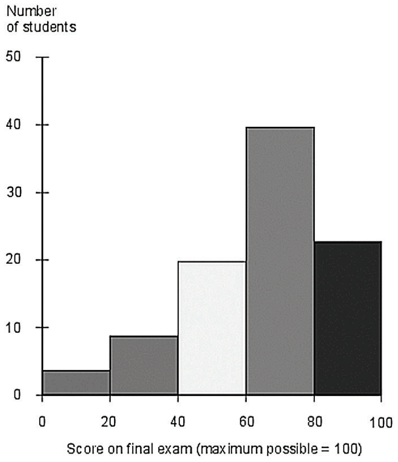
b. Double or compound bar graphs

These are used to make a comparison of various related data items like comparing expenses of the various departments in our government.



c. Histogram

A histogram is used to show grouped data. One bar's data follows directly from the previous one.



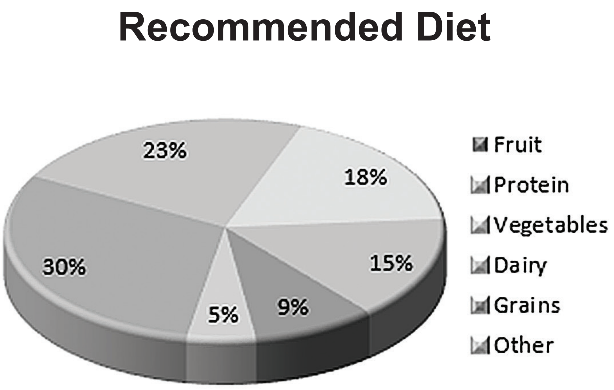
For example:

Notice that 20-40 follows on immediately from 0-20 and 40-60 follows on from 20-40 so on

d. Pie Chart

A pie chart is a circle that is divided into slices where the size of the slice represents the size of the data.

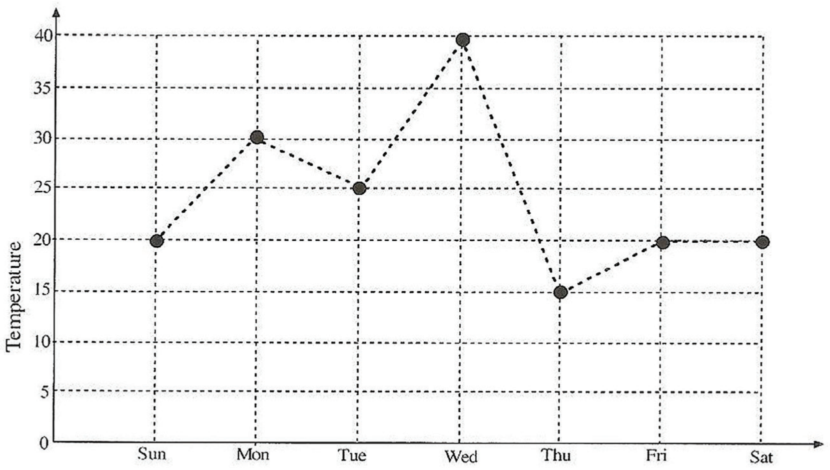
For example:



This pie chart represents the types of foods a person should eat in their diet. At a glance, you can easily see that 'fruit' has the largest percentage and that 'other' has the smallest.

e. Broken Line Graph

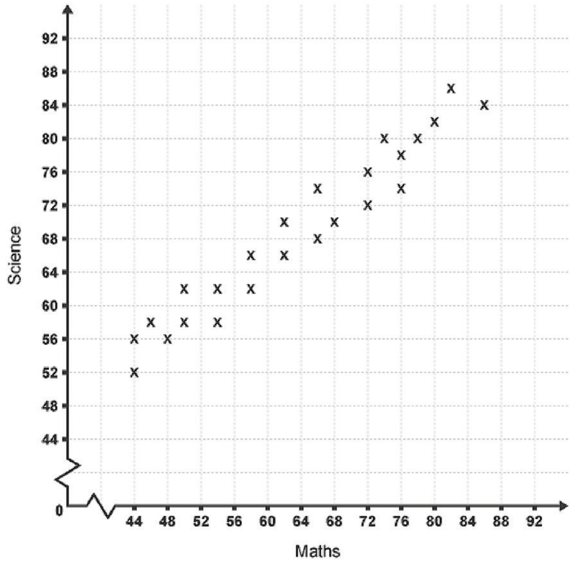
A broken line graph shows change. Broken line graphs represent information collected at different points. They do not contain information about situations in between these points. The following graph represents the temperatures recorded at 13h00 each day from a particular Sunday to the following Saturday.



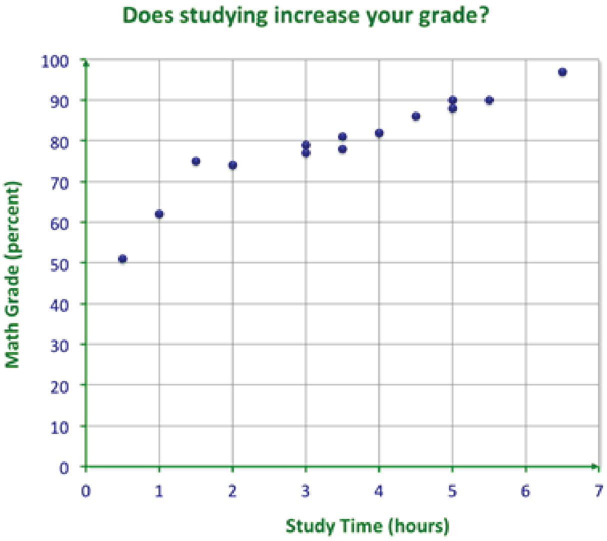
f. Scatter Plots

NOTE: This is new to Grade 9 so more time should be spent on this. The previous graphs have all been covered before.

A scatter plot represents two sets of data which may or may not be connected in some way. For example, a scatter plot could be made of one class' mathematics and science marks. Each point would represent one learner's maths and science mark. One subject would be represented on the horizontal axis and the other would be represented on the vertical axis.



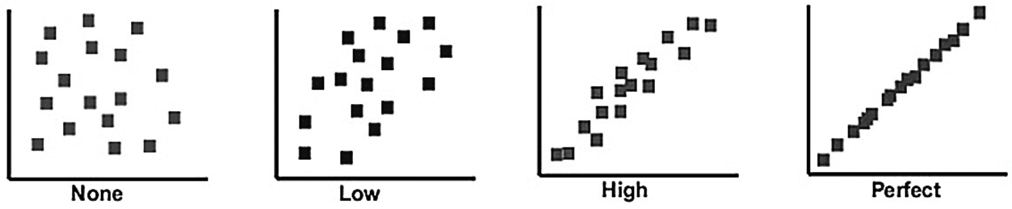
Notice that the mark closest to the bottom and left represents a learner that got 44% for maths and 52% for science.



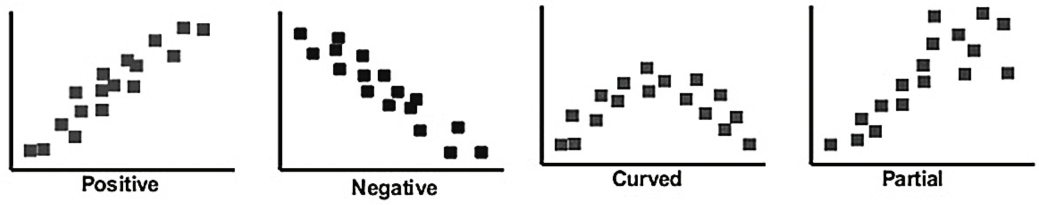
This scatterplot would be to try and determine if there was a correlation between the amount of hours studied and a mathematics result.

To find if there is a correlation (establishing whether there is a connection) between the sets of data represented the following applies:

**Degrees of correlation**



**Degrees of correlation**



Using these rules, we can see that there is a strong (high) positive correlation between a learner's mathematics marks and their science marks.
Notice that the points also form a line – this tells us that the correlation is also linear.

Similarly with the number of hours studied and the maths result.

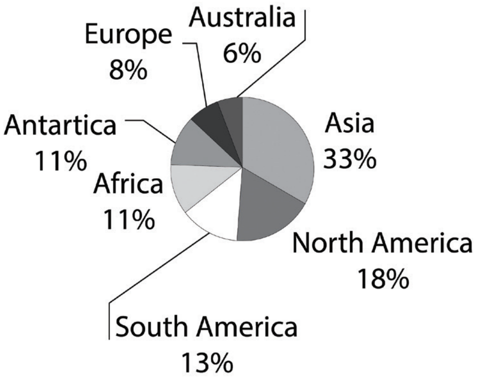
## Interpreting, Analysing and Reporting Data

1. Interpreting data means to look at the data that has been collected and represented and consider what it all means.
2. It is at this stage that consideration needs to be given to whether the data could be misleading (has it been presented in a way that could give the wrong impression?) or if it has been manipulated (has it been changed in order to mislead?).
3. Changing the scale of the axes on a graph or changing the class intervals on a histogram can lead to data not being represented correctly.
4. When reporting on the results of a research the following information needs to be considered: An explanation needs to be given discussing:
   - the purpose of the research
   - the population and how a sample was taken
   - the way the data was collected
   - what conclusions you have come to

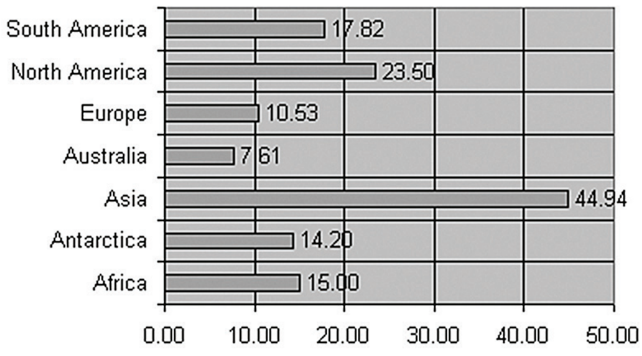
5. It is also useful to try and :
    - make predictions based on the data collected
    - discuss any limitations that may have affected the study.
6. To develop critical analysis skills, the following types of activities should be used:
    - Learners should compare the same data represented in different ways e.g. in a pie chart or a bar graph or a table, and discuss what information is shown and what is hidden; they should evaluate which form of representation works best for the given data.

For example:
The surface area of the continents has been represented in a pie chart and a bar chart. Note how useful they both are but that they show the information a little differently. The pie chart enables one to, at a glance, see which continents take up the most or least amount of space. The bar chart on the other hand gives actual measurements in millions of square kilometres which is not seen in the pie chart. A discussion along similar lines amongst learners will be worthwhile.

**Surface Area of Continents**



**Surface Area of Continents**

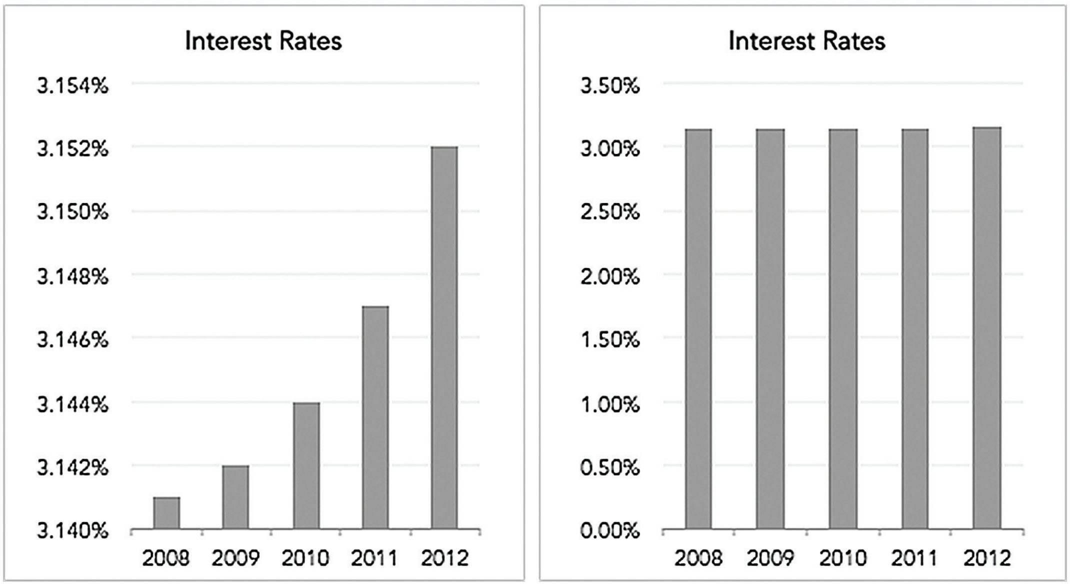


Surface area in millions of sq. km

- Learners should compare graphs on the same topic but where data has been collected from different groups of people, at different times, in different places or in different ways. Here learners should discuss differences between the data with an awareness of bias related to the impact of data sources and methods of data collection on the interpretation of the data.
- Learners should compare different ways of summarizing the same data sets, developing an awareness of how data reporting can be manipulated; they should evaluate which summary statistics best represent the data.
- Learners should compare graphs of the same data, where the scales of the graphs are different. Here learners should discuss differences with an awareness of how representation of data can be manipulated; they should evaluate which form of representation works best for the given data.

For example:

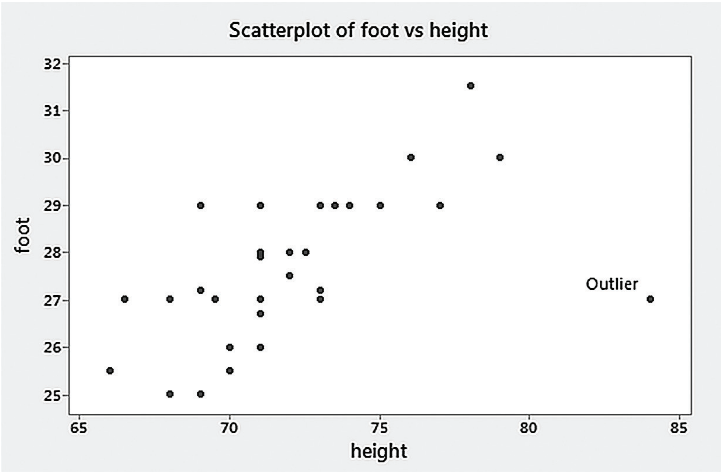
**Same Data, Different Y-Axis**



Note how different these graphs look even though they represent the same data and are both accurate. A discussion with learners with regards to how the scale can distort data is an important one

- Learners should compare data on the same topic, where one set of data has extremes or outliers, and discuss differences with an awareness of the effect of the extremes or outliers on the interpretation of the data. In particular, extremes affect the range and outliers which are identified on scatter plots.

  For example:



Notice that in the scatterplot above, most of the points are clustered in one area. The outlier represents a very tall person who has a fairly small foot.

- Learners should write reports on the data in short paragraphs

# TOPIC 4: PROBABILITY

## INTRODUCTION

- This unit runs for 4.5 hours.
- It is part of the content area, 'Data Handling' and counts for 10% in the final exam.
- It is important to note that having a good understanding of probability is one way to think about your world and the decisions you make every day. In living our lives, we often take on risks and expose ourselves to dangers (not necessarily physical but perhaps losing money for example). We may try things which we think will probably succeed, but we're not really sure. Probability theory gives us a way to think about these decisions and may help to take control of them.

## SEQUENTIAL TEACHING TABLE

| INTERMEDIATE PHASE/GRADE 8 | GRADE 9 | GRADE 10/FET PHASE |
|---|---|---|
| LOOKING BACK | CURRENT | LOOKING FORWARD |
| Consider a simple situation (with equally likely outcomes) that can be described using probability and:<br>• List all the possible outcomes<br>• Determine the probability of each possible outcome using the definition of probability<br>• Predict with reasons the relative frequency of the possible outcomes for a series of trials based on probability<br>• Compare relative frequency with probability and explain possible differences | Consider situations with equally probable outcomes and:<br>• Determine probabilities for compound events using two-way tables and tree diagrams<br>• Determine the probabilities for outcomes of events and predict their relative frequency in simple experiments<br>• Compare relative frequency with probability and explain possible differences | • Compare the relative frequency and theoretical probability of the outcome.<br>• Venn diagrams, contingency tables, tree diagrams<br>• Mutually exclusive and complementary events.<br>• The identity for any two events A and B: P(A or B) = P(A) + (B) – P(A and B)<br>• Dependent and independent events.<br>• The counting principle. |

# GLOSSARY OF TERMS

| Term | Explanation / Diagram |
|---|---|
| **Probability** | The likelihood or chance of something happening.<br>It is usually expressed as a fraction (but could also be as a decimal or percentage).<br>A probability answer is ALWAYS in the following range: $0 \leq x \leq 1$<br>(ie answer can only be zero, one or a common fraction). |
| **Trial/Experiment** | The process of trying something out to find the chance (probability) of an event occurring.<br>For example: Tossing a coin 100 times. |
| **Outcome** | A possible result from an experiment<br>For example: 'tails' is one of two possible outcomes when tossing a coin. |
| **Experimental probability** | The result of doing an experiment to find the chances of an event occurring.<br>For example: An experiment was conducted to see how many tails appeared when a coin was tossed 100 times. The result was $\frac{47}{100}$. |
| **Relative Frequency** | The outcome of an experiment. In the above example $\frac{47}{100}$ is the relative frequency. |
| **Theoretical probability** | The probability of an event happening using knowledge of numbers.<br>For example: The chance of getting tails when tossing a coin is  because 'tails' is the outcome (one) being considered from a possible two outcomes.<br>It is the actual calculation of the probability of an event happening.<br>$\text{probability of an event P(A)=}\frac{\text{Number of times the event has occured [N(A)]}}{\text{Total Number of possible outcomes [N(S)]}}$ |
| **Tree diagram** | One method used for counting the number of possible outcomes of an event is to draw a tree diagram. The last column of the tree diagram shows all of the possible outcomes. The list of all possible outcomes is called the sample space, and a specific outcome is called an event. |
| **Two way table** | When two experiments are happening at the same time we can use a two way table. |

# SUMMARY OF KEY CONCEPTS

An important concept of probability that needs to be understood from the beginning is that no answer to a probability question can ever be less than zero or bigger than one.

Every answer will always lie somewhere from zero to 1. Therefore, most answers are fractions except those that are actually zero or one.
Remember that fractions can be written in more than one way.

For example,

$$\frac{1}{2} = 0,5 = 50\%$$

If there is an equally likely chance of an event happening (also known as a 50/50 chance), any one of the above fractions would represent the probability of such an event occurring. (Remember that 50% means 50 divided by 100 and is therefore not a number bigger than one although it may at first appear to be so)

$$P(event) = \frac{the\ number\ of\ ways\ the\ event\ could\ happen}{the\ total\ number\ of\ possible\ equally\ likely\ outcomes}$$

What each bit means:
P (event) – this is just a short way of writing: “the probability of an event happening”.

For example:
P (even number) means “the probability of getting an even number”

The number of ways the event could happen – you have to carefully count up all the different ways there are of the event you are interested in actually occurring. For example, if you roll a die and want to get an even number, the number of ways would be 3 as there are 3 even numbers on a die (2, 4 and 6)
The total number of possible equally likely outcomes – you must carefully count up all the total possible things that could happen, but you must remember that they must all be equally likely! For example, if you roll a die, there are 6 possible outcomes.

NOTE: In Grade 9 probability we only deal with equally likely events. This will change in Grade 10

The answer will be a fraction. In the above case (rolling a die and wanting to get an even number) the probability would be:

$$\frac{3}{6} = \frac{1}{2}$$

**Teaching Tip:** Although it is preferable that learners simplify their answers, it is important that we, as teachers, remember that you are trying to teach them probability at this stage not fractions. Rather focus on this as a new concept, so if learners don't simplify then don't penalise them.

A worked example:
There are all the letters of the alphabet placed in a bag. A person closes their eyes and chooses one letter from the bag. What is the probability that the person picks out:

a. a vowel?

b. a letter?

c. a number?

d. the initial of his/her first name?

a. The number of vowels is 5. The total number of letters in the alphabet is 26
$\therefore$ P(vowel)= $\frac{5}{26}$

b. The number of letters in the bag is 26. The total number of letters in the alphabet is 26
$\therefore$ P(a letter)= $\frac{26}{26}$ =1
In other words it is certain that a person would get a letter when picking out of a bag that only has letters in it.

c. The number of numbers in the bag is zero. The total number of letters in the alphabet is 26
$\therefore$ P(a number)= $\frac{0}{26}$ =0
In other words, there is zero chance of picking a number out of the bag that only has letters in it.

d. The number of letters in the bag that are the start of the person's own name is one. The total number of letters in the alphabet is 26
$\therefore$ P(own initial)= $\frac{1}{26}$

Points to remember regarding probability:

- If something has a probability of 1, it is CERTAIN to happen
- If something has a probability of 0, it is IMPOSSIBLE
- All probabilities lie between 0 and 1, so if you find yourself with a negative answer, or something like 2.4, then you have done something wrong!!!

## Relative frequency and probability

Ensure that learners understand the difference between relative frequency and probability. Relative frequency could change every time you do an experiment. If different people were doing the same experiment, the relative frequency results could be different.

Probability however will be the same as it is theoretical.
For example, If a coin is tossed 50 times by two different people, it is likely that they will not get the same results. Person A could get 19 tails and 31 heads while Person B could get 28 tails and 22 heads.

Person A would say that there is a $\frac{19}{50}$ chance of getting a tail while Person B would argue that there is a $\frac{28}{50}$ $\left(\frac{14}{25}\right)$ chance of getting a tail.

Theoretically, the probability of getting a tail is $\frac{1}{2}$ as there should be a 50% chance of getting one of the two possible outcomes.
The more times you repeat the activity involved in the experiment the closer the relative frequency should be to the probability. (Tossing a coin 500 times is likely – note the probability word here - to produce a result of getting tails very close to $\frac{1}{2}$ whereas tossing the coin only 10 times could produce a result of $\frac{7}{10}$ )

NOTE: Additional activities in which learners engage in experiments so that they can compare probability and relative frequency of the experiment are important in order to further their understanding of the difference between the two concepts. Investigations are advised so that different learners can do as many trials as possible or results of different trials are combined to see if there is any relationship/correlation of/between the number of trials and relative frequency.

There is an investigation in the resource section at the end of the topic that can be used if the textbook being used at your school does not have one in.

## Probability of an event NOT happening

A key concept towards understanding probability is that the sum of all the possible outcomes of an event will be 1.

If a coin is tossed, there is a $\frac{1}{2}$ chance of getting a heads. There is also a $\frac{1}{2}$ chance of getting tails. These are the only two possible outcomes.

(Note: it is better to say a 1 out of 2 chance rather than say there is half a chance. This encourages learners to be thinking of the numerator and denominator separately)

$$\frac{1}{2}+\frac{1}{2}=1$$

For example: You have 5 t-shirts in a drawer. One of them is red.

a. What is the probability of pulling out a red t-shirt?

b. What is the probability of not pulling out the red t-shirt?

a. The number of red t-shirts in the drawer is one. The total number of t-shirts in the drawer is 5.

$\therefore P\,(red)=\frac{1}{5}$

b. The number of no red t-shirts in the drawer is four. The total number of t-shirts in the drawer is 5.

$\therefore P(not\ red)=\frac{4}{5}$

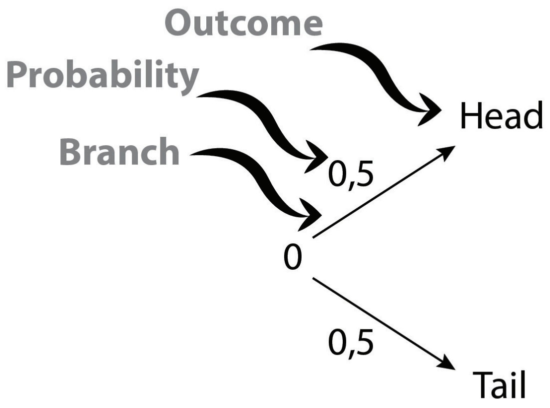
BUT, to use the statement above we could also calculate this as follows:

$1-\frac{1}{5}=\frac{4}{5}\quad because\quad \frac{1}{5}+\frac{4}{5}=1$ (the sum of all the possibilities is 1)

## Probability of compound events

1. Tree diagrams

A tree diagram is simply a way of representing a sequence of events. Here is a tree diagram for the toss of a coin:

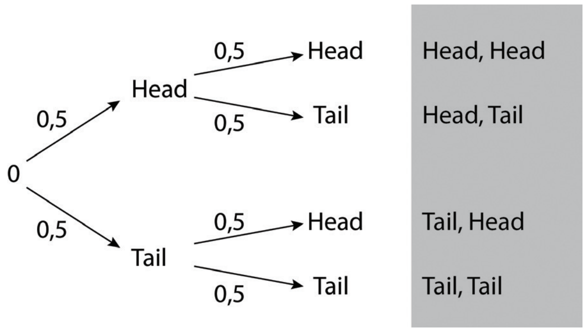


There are two "branches" (Heads and Tails)

- The probability of each branch is written on the branch
- The outcome is written at the end of the branch

**Teaching Tip:** Take the time to stress to learners what they write on the branch and what they write at the end of the branch. Many learners do not get this right.

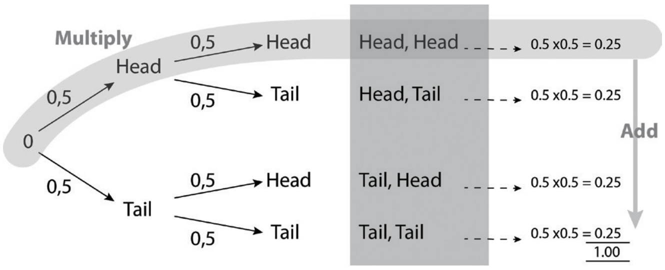
The tree diagram can be extended to show the tossing of a coin a second time:



Notice the four possible outcomes of tossing a coin twice.

The tree diagram can now be used to find probabilities of different outcomes. (Remember that there are 4 possible outcomes)

In order to calculate overall probabilities, we multiply going along the branches that lead to the outcome wanted. If there are two branches that lead to a preferred outcome (such as wanting to get two results the same – 2 heads or 2 tails) then these answers will be added.



For example:
The probability of getting two heads is: 0,5 x 0,5 = 0,25 or $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$
(notice the 4 – there is a 1 in 4 chance of getting 2 heads)

The probability of getting at least one head is:
First look at all the outcomes. How many of these outcomes have at least one head in them? 3 of them do (only the last one has no heads at all). We will have to first multiply along the branches that lead to each of these three outcomes, then add all of those answers together.

$$(\frac{1}{2}\times\frac{1}{2})+(\frac{1}{2}\times\frac{1}{2})+(\frac{1}{2}\times\frac{1}{2})=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{3}{4}$$

Or (0,5 x 0,5) +(0,5 x 0,5) +(0,5 x 0,5) = 0,25 + 0,25 +0,25 = 0,75

Summary of concepts used in the above example:

- To find the probability of something happening AND something else happening, just multiply your probabilities together
- To find the probability of something happening OR something else happening, just add up your probabilities

2. Two-way tables

When you have two experiments happening at the same time (like two coins being tossed), the safest way to ensure you account for all of the outcomes is to draw up a two way table.

| | **Head** | **Tail** |
|---|---|---|
| **Head** | H-H | H-T |
| **Tail** | T-H | T-T |

We can now see that there are 4 equally likely outcomes, and so to get our probabilities, we just need to count up the number of outcomes we are interested in.

For example, the probability of getting a head and a tail is:

$P\left(head\ and\ tail\right)=\frac{2}{4}=\frac{1}{2}$

(2 of the 4 outcomes represent a head and a tail. These are H-T and T-H )

A two-way table does not just have two rows and two columns like the previous one. Two-way means that information will be read from the table by looking down and across rows and columns (a little bit like reading a coordinate in a Cartesian plane)

In the next example, the two-way table has 6 rows and six columns. It represents throwing two dice (Die is singular and dice is plural)

It shows all the outcomes if we were to add the numbers we get on each of the die.

| | **1** | **2** | **3** | **4** | **5** | **6** |
|---|---|---|---|---|---|---|
| **1** | 2 | 3 | 4 | 5 | 6 | 7 |
| **2** | 3 | 4 | 5 | 6 | 7 | 8 |
| **3** | 4 | 5 | 6 | 7 | 8 | 9 |
| **4** | 5 | 6 | 7 | 8 | 9 | 10 |
| **5** | 6 | 7 | 8 | 9 | 10 | 11 |
| **6** | 7 | 8 | 9 | 10 | 11 | 12 |

A pair of dice are rolled. The total on the 2 die are added.

a. What is the probability of getting a total score of 5?

b. What is the probability of getting a total score of 1?

c. What is the probability of getting a total score greater than 10?

a. Check how many scores of 5 are possible. Note that there are 4. The total number of outcomes recorded is 36.

$$P(\textit{total of } 5) = \frac{4}{36} = \frac{1}{9}$$

b. Check how many scores of 1 are possible. Note that there are zero. The total number of outcomes recorded is 36.

$$P(\textit{total of } 1) = \frac{0}{36} = (0)$$

c. Check how many scores there are that are bigger than 10 (all the 11's or 12's in this case). There are 3. The total number of outcomes recorded is 36

$$P(\textit{greater than } 10) = \frac{3}{36} = \frac{1}{12}$$

# RESOURCES

Name:___________________________

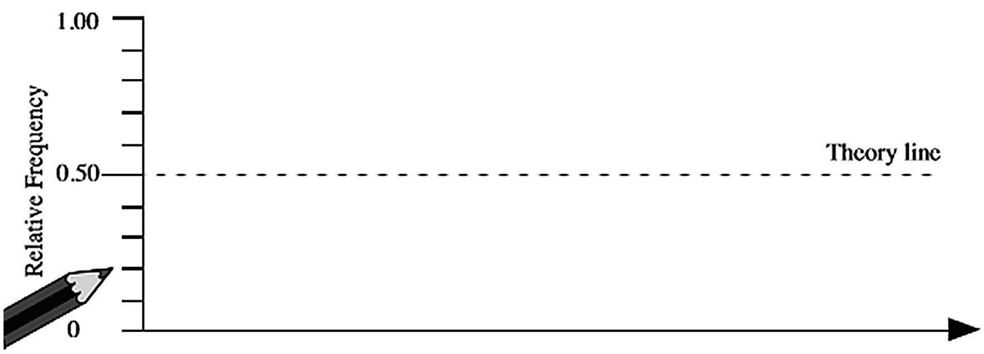
### Experimental Probability and Relative Frequency Investigation

Flip a coin 30 times and fill in the tables and graph below

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Throw | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Result | | | | | | | | | | |
| Throw | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| Result | | | | | | | | | | |
| Throw | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| Result | | | | | | | | | | |

Remember: relative frequency = $\frac{number\ of\ tails}{number\ ofthrows}$

| After… | 5 Throws | 10 Throws | 15 Throws | 20 Throws | 25 Throws | 30 Throws |
|---|---|---|---|---|---|---|
| Number of Tails | | | | | | |
| Relative Frequency | | | | | | |



number of throws